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7

Detecting Periodicity in Quantitative versus Semi-Quantitative Time Series

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Ordered categorical (or *semi-quantitative*) data are frequently encountered in ecology (e.g., Steen et al. 1990; Ménard et al. 1993). Researchers often resort to semi-quantitative measures (to describe abundance patterns, age or stage structures, environmental factors, etc.) to reduce processing time and/or because of financial constraints, while retaining an acceptable level of accuracy.

Time series analysis is a powerful tool for detecting underlying patterns and mechanisms in time-ordered data and is being employed increasingly in ecology (Platt and Denman 1975; Shugart 1978; Weber et al. 1986; Rose and Leggett 1990). Nearly all "traditional" methods of time series analysis require that data be from a continuous state space (from here on referred to as *quantitative data*). On the other hand, semi-quantitative data are from a finite set and cannot be analyzed using traditional methods. There are several techniques that deal with semi-quantitative time series, but these are primarily concerned with assessing short-term dependence between observations (e.g., Markov chain models). Few techniques exist for detecting periodicities. There is a body of theory for binary time series analysis (Kedem 1980), but techniques to explore data with more than two categories are less well developed.

Here we briefly review a technique for dealing with time-ordered, semi-quantitative data—the contingency periodogram as developed by Legendre and Legendre (1979) and as further explored by Legendre et al. (1981). We explore the potential change in ability to detect periodicities when the form of a dataset changes from quantitative to semi-quantitative (five and two categories), and how these changes may affect interpretation of results.

Our approach is to analyze four datasets. Two of these are simulated models, one of a quasi-periodic population, the other of a truly chaotic population. These models reflect the growing interest in chaos in ecology. The other two datasets consist of a horizontal temperature profile from Lake Ontario, and of recorded abundance of a zooplankter at a Mediterranean reference station. The first three datasets are quantitative, and thus allow comparison between traditional time series methods and the contingency periodogram (after categorizing). The last dataset was collected as semi-quantitative data and thus can be analyzed only by means of the contingency periodogram.

Methods

Traditional Time Series Analysis

When time series are sufficiently long, and data are quantitative, time series analysis in the traditional sense (e.g., Chatfield 1989; Diggle 1990) can be a powerful tool. The reader is urged to examine the abundant literature on this subject for a more complete understanding of these techniques.

The estimated spectrum of a time series partitions the variance of the series into its component frequencies (time^{-1}) or wavenumbers (length^{-1}). Peaks in the spectrum indicate what scales are contributing the most variance to the series. In this manner, periodicities, if present, are detected. The estimated spectrum is calculated by smoothing the raw periodogram. For our analyses, we have calculated the raw periodograms of the datasets in their quantitative form, *but we have not smoothed them*. This facilitates direct comparison with the contingency periodogram of the datasets in their semi-quantitative forms.

The Contingency Periodogram

The contingency periodogram (Legendre et al. 1981) is one method that can potentially identify periodicities in short, semi-quantitative time series with two or more categories. As with other time series methods, the data must be stationary (constant mean and variance) and detrended.

The contingency periodogram is based on contingency tables. To illustrate this method, we give an example taken directly from Legendre et al. (1981). Table 7-1 shows a time series of $N=16$ observations. Each observation can be one of three states (categories). The series is divided into nonoverlapping subseries of period T, which can range from 2 to $N/2$. In the example, T ranges from 2 to 8. A contingency table is then constructed for each period T. The rows of the contingency table correspond to the states of the data. The columns are the sequential observations in period T. The values in the contingency table are the number of observations in a given state S of the process (row) made at one observation of the period T (column). Table 7-2 shows the contingency table for $T=5$ in the example time series. The value of 3 in Row Three (state 1), Column Two (observation 2) signifies that there are a total of three state 1's located in the second position when the series is divided into nonoverlapping subseries of $T=5$ (positions 2, 7, and 12 in Table 7-1).

Table 7-1. Example time series of 16 observations that can each be one of three states (from Legendre et al. 1981).

States	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	Totals
3				X	X				X					X	X		5
2			X			X		X		X			X				5
1	X	X					X				X	X				X	6

Table 7-2. Contingency table for the period T = 5, for the example time series in Table 7-1 (from Legendre et al. 1981).

States	Observation axis (X) 1	2	3	4	5	Total
3	0	0	0	3	2	5
2	1	0	3	0	1	5
1	3	3	0	0	0	6
Total	4	3	3	3	3	16

For each of the T contingency tables, the entropies H (uncertainties) are estimated following information theory:

$$H(S) = -\sum_{i=1}^{s}\frac{N_i}{N}\log\frac{N_i}{N} \tag{1}$$

$$H(X) = -\sum_{j=1}^{T}\frac{N_j}{N}\log\frac{N_j}{N} \tag{2}$$

$$H(SX) = -\sum_{i=1}^{s}\sum_{j=1}^{T}\frac{N_{ij}}{n}\log\frac{N_{ij}}{N} \tag{3}$$

where $H(S)$ is the uncertainty of the states, $H(X)$ is the uncertainty of the observation axis, $H(SX)$ is the uncertainty of the entire contingency table, N_{ij} is the value in row i and column j of the contingency table, N_i and N_j are the totals of row i and column j, respectively, and s is the number of states. The contingency statistic is:

$$H(S \cap X) = H(S) + H(X) - H(SX). \tag{4}$$

Simply put, $H(S \cap X)$ is a measure of how much uncertainty in the states (categories) is explained by a given period T.

If the rows and columns are independent, the quantity $2N*H(S \cap X)$ is asymptotically distributed as χ^2 with $(s-1)(T-1)$ degrees of freedom when $\log_e$ is used (Kullback 1959). However, as Legendre et al. (1981) point out, each value of the contingency periodogram (i.e., $H(S \cap X)$ for each period T) is not independent, and therefore one cannot test each value for significance. Instead, if a value is below its critical value, then it is definitely too small to be different from zero.

If the value exceeds its critical value, it still may not be different from zero, "but their ecological implications may be considered" (Legendre et al. 1981). Figure 7-1 shows the contingency periodogram for the example series in Table 7-1. A peak is evident at the period $T = 5$.

The number of states (categories) and their definitions are clearly important. However, the contingency periodogram does not take into account the ordering of the states, and, as a result, one may be less certain of identifying actual periodicities in semi-quantitative data sets (Legendre et al. 1981). Despite this potential problem, results form Legendre et al. (1981) and from this study indicate that the contingency periodogram is a useful method for detecting periodicities in semi-quantitative time series. Legendre et al. (1981) propose a method for dividing rank-ordered variables into states when there are no available ecological or biological criteria. In our analyses, we have chosen to use five and two categories for all of our examples. Five was chosen to match the original number of categories in one of our datasets (salp abundance), and two was chosen arbitrarily as some value less than five to generate decreasing resolution in the data. In the salp example, we use ecological criteria for establishing the partition

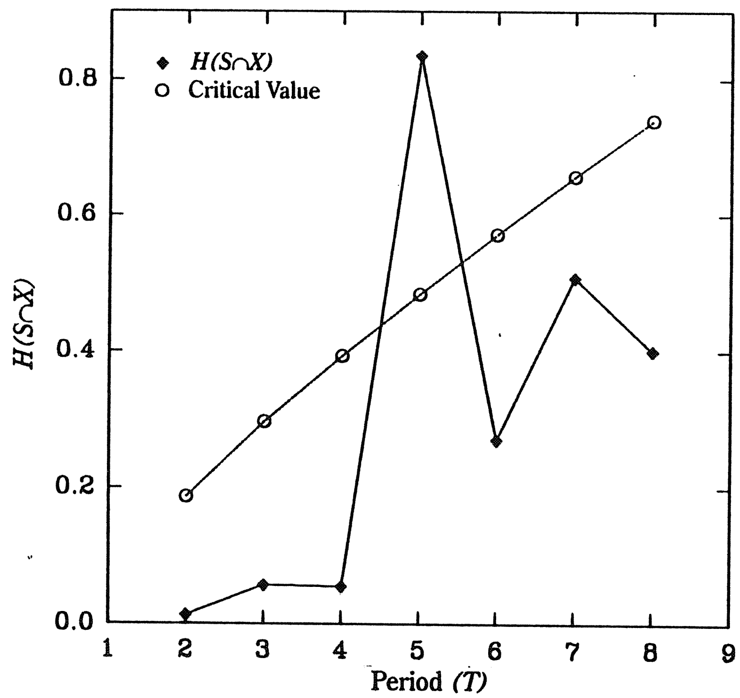

Figure 7-1. The contingency periodogram for the time series in Table 7-1 (from Legendre et al. 1981). Critical values are $\chi^2_{(s-1)(T-1)}/2N$ where $s = 3$ and $N = 16$.

values of the binary series. Otherwise, series are partitioned to give near-equal numbers of data points in each category, or are partitioned based on an approximate $\log_2$ scale. In each example, we identify how data are partitioned.

Also note that this method allows missing values, which are usually the rule in long-term ecological studies. For a more rigorous development of the contingency periodogram, see Legendre et al. (1981).

Analyses

Example 1: A Seasonal Model of Population Dynamics

We generated a time series from a population dynamics model developed by Burkey and Stenseth (in press). The model population is territorial during summer months and nonterritorial during winter months. The population does not reproduce during the winter, which introduces seasonal and strong yearly patterns. We chose parameters to generate a quasi-periodic series. For details of the model, see Burkey and Stenseth (in press).

Figure 7-2 shows a 100-year (stationary) time series of the model as (A) quantitative data, (B) quantitative data partitioned into five ordered categories, and (C) quantitative data partitioned into an ordered binary series. Samples are taken 10 times per year, giving a total of $N = 1000$ data points. The partition values are 11, 25, 45, and 80 for the five categories. This represents an approximate doubling of the category ranges and also provides a nearly equal number of points in each category (although there are fewer points in the last category). In the binary case, the 0's consist of the first two classes (≤ 25) in the five-category case, and the 1's consist of the last three classes (> 25) in the five-category case.

The raw periodogram (Figure 7-3) reveals the strong annual pattern (10 time steps), as well as the semiannual and seasonal patterns, despite the quasi-periodic nature of the model. The presence of a number of smaller peaks at other frequencies indicates harmonics or other possible dynamics, but these cannot be considered significant, as the large peaks are several orders of magnitude greater.

Figures 7-4A and B show the contingency periodograms for both semi-quantitative series. Annual and semi-annual periodicities ($T = 10$ and $T = 5$, respectively, plus harmonics) are detected in both cases. However, a quarterly cycle that exceeds its critical value in the five-category series (Figure 7-4A) does not exceed its critical value in the binary series (Figure 7-4B). This is expected: as the resolution of the data decreases, the ability to detect finer-scale periodicities also decreases. Thus, it would appear that five categories is a sufficient number to detect periodicities down to a seasonal level, while (presumably) also reducing costs associated with collecting and processing quantitative measures.

It is worth noting that, when examining the contingency periodogram for values that exceed their critical values, we are performing multiple tests. Typically

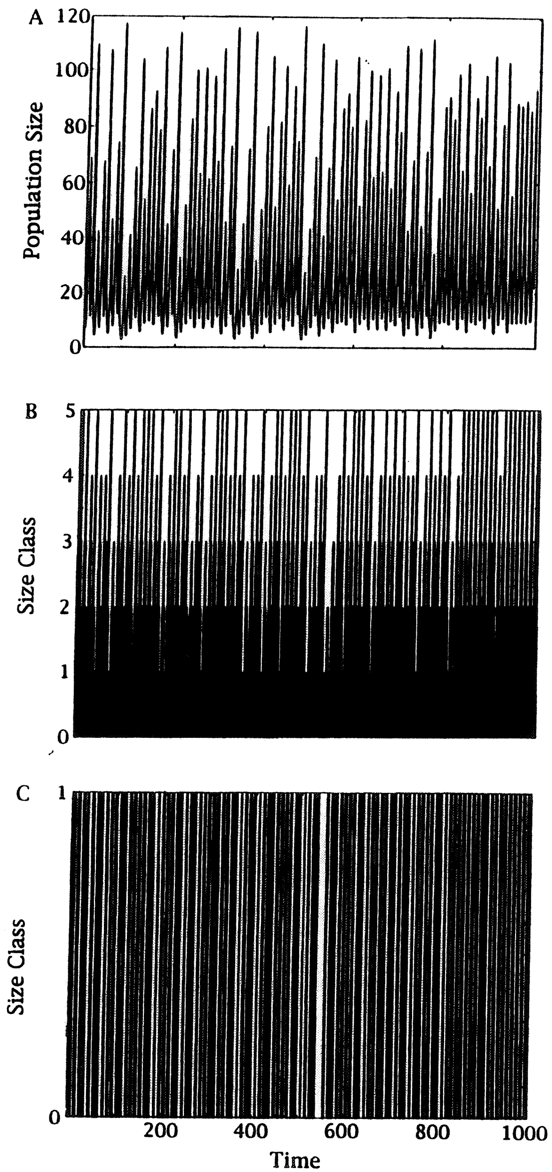

Figure 7-2. Time series generated from a seasonal model of population dynamics (Burkey and Stenseth, in press) as: (A) quantitative data, (B) quantitative data partitioned into five groups, and (C) quantitative data partitioned into two groups. Parameters of the model were chosen to generate a quasi-periodic series. Ten time units equals one year.

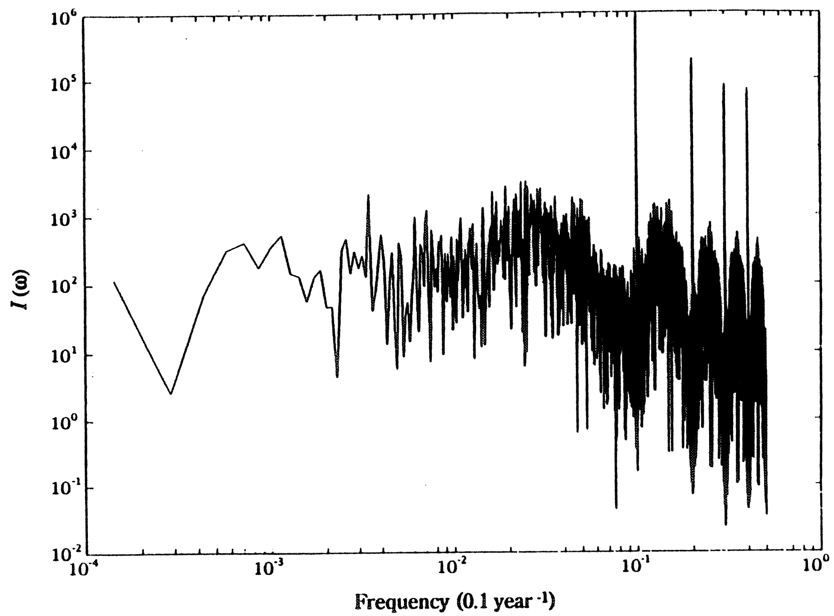

Figure 7-3. Raw periodogram of quantitative time series shown in Figure 7-2A.

one should apply a Bonferroni correction by dividing the α-value (0.05) by the number of tests (in this case, T = 2 to 100, or 99). However, a Bonferroni approach will not work when the number of comparisons (T) gets large (possibly at $T \geq 10$; A. Solow, pers. comm.). It may be possible to develop a portmanteau test for global significance based on randomization. If any of the contingency periodogram values exceeds this portmanteau critical value, then independence would be rejected.

Example 2: A "Super" Chaotic Model

A chaotic signal represents extreme deterministic, nonlinear behavior: it is aperiodic and gives a flat spectrum. Thus, it is unpredictable on a long-term basis (May 1974; Parker and Chua 1987).

A chaotic time series was generated using the simple logistic equation:

$$X(t+1) = abe^{-bX(t)/m}. \quad (5)$$

This model was studied by Vandermeer (1982a,b) and resembles the population fluctuations of rare species. For example, for an insect population X growing on plant seeds, the model gives the number of insects at generation t + 1 given the number of insects at previous generation t under the following assumptions: (1)

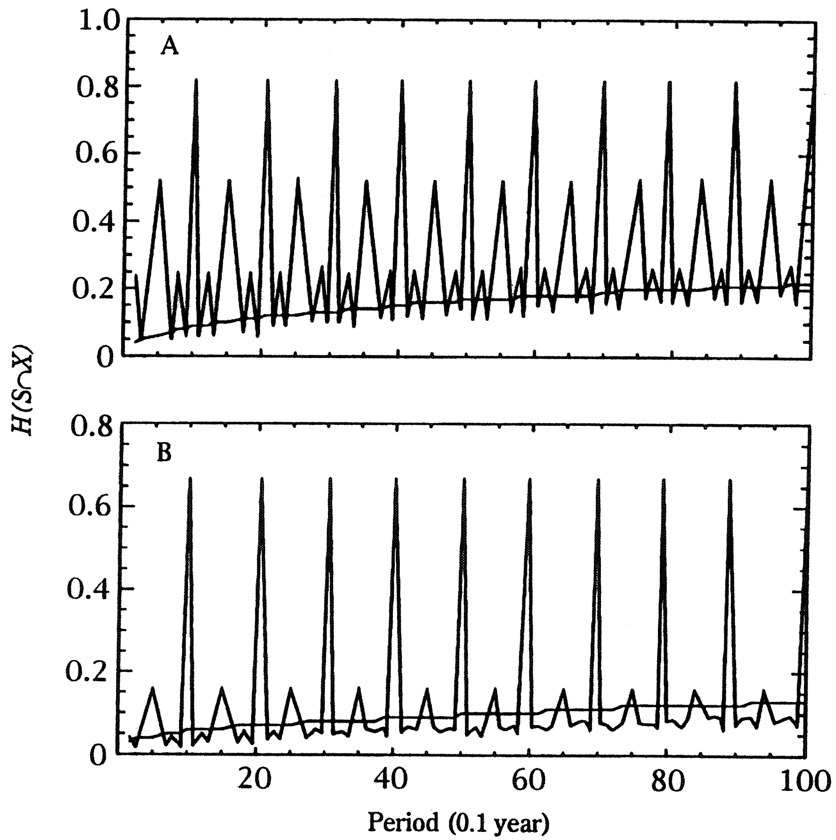

Figure 7-4. Contingency periodograms of semi-quantitative time series shown in Figures 7-2B and 7-2C, respectively: (A) five categories and (B) two categories.

any individual insect lays an average of b eggs at random, (2) only one larva can survive on a single seed with survivorship rate a, and (3) the total number of available seeds is m.

Vandermeer (1982a,b) showed that for very large values of the product ab (i.e., large growth rate), the model enters a particular region of dynamical behavior called "super" chaos. This behavior is characterized by a stable linear relationship between the numbers of years of low numbers in a population n and the amplitude of the population burst X_b. Thus, there exists the possibility of predicting n from X_b (or, in Vandermeer's words, to "resolve" the chaotic behavior).

A time series of $N = 1000$ points (Figure 7-5) was generated by iterating this "super" chaotic version of equation (5). Figure 7-5 also shows the population X of the time series partitioned into semi-quantitative measures (five and two categories). The categories were classified using a geometric progression with base 2 for the five-category case. The binary series was partitioned at the value separating the third and fourth categories in the five-category series.

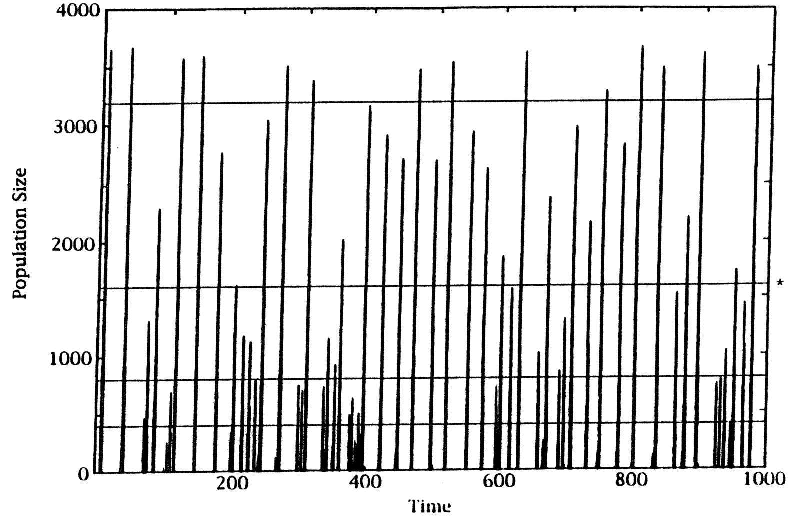

Figure 7-5. Time series generated from the "super" chaotic model in equation (5) (Vandermeer 1982a,b). Solid lines indicate partitioning for five categories. Asterisk indicates partitioning for two categories.

Despite the "resolution" property shown by Vandermeer (1982a,b), the raw periodogram of this time series (not shown) appears to be random (i.e., a flat periodogram), as expected. The contingency periodogram of the five-category series (Figure 7-6A) reveals no values different from zero, exemplifying the aperiodic nature of the signal. A similar result is found in the binary case (Figure 7-6B). Thus, the analyses for the quantitative form and both semi-quantitative forms of this "super" chaotic time series give similar results: no detectable periodicities.

Example 3: Application to Limnological Data

Figure 7-7 shows temperature for a horizontal transect at 30-m depth from Lake Ontario in November 1991. The data were collected with a conductivity-temperature-depth (CTD) probe mounted with other instruments on an Endeco V-fin (Sprules et al. 1992). The sampling interval between data points is 1 second, or approximately 2.1 m. If we assume temporal homogeneity, then we can investigate the periodicities of the spatial structure (in one dimension) of the data.

The data were detrended with a linear regression, and the raw periodogram

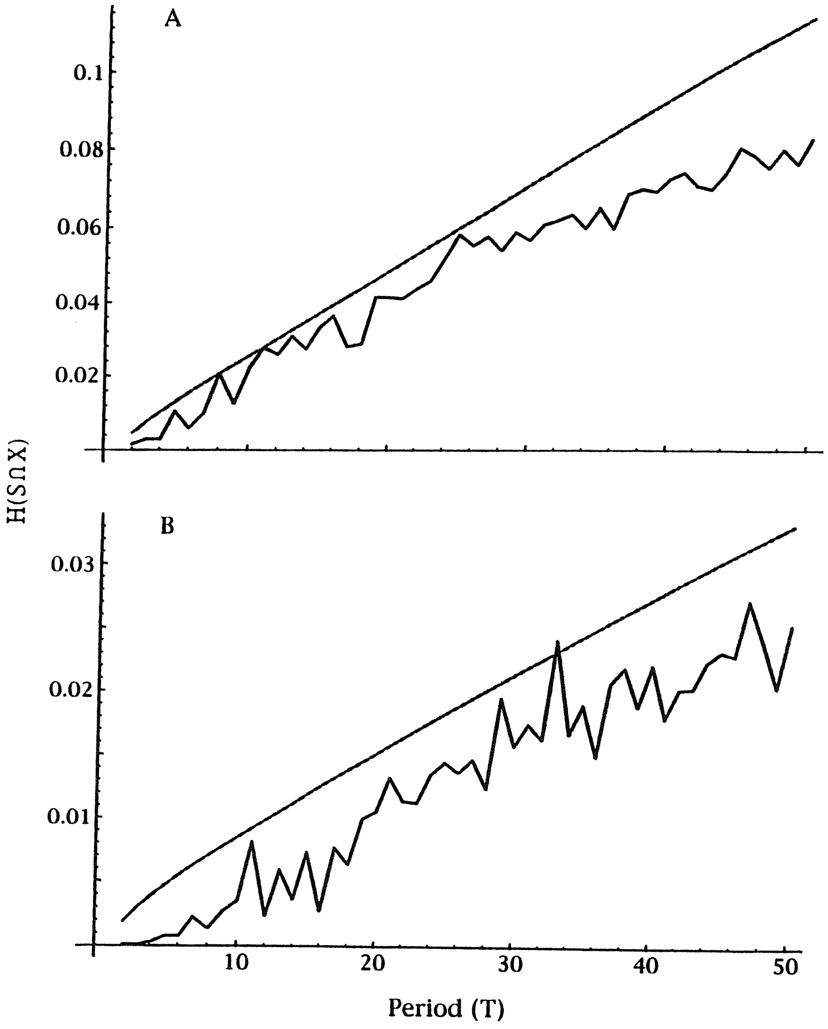

Figure 7-6. Contingency periodograms of (A) five-category and (B) two-category time series generated from the "super" chaotic model of equation (5) shown in Figure 7-5.

was calculated for the residuals. The residuals were then partitioned into semi-quantitative series (five and two categories). The categories were determined by ensuring a roughly equal number of data points in each category for both semi-quantitative series.

The raw periodogram (Figure 7-8) shows a dominating, long-term trend in the temperature, evident at the lowest wavenumbers (largest spatial scales). The

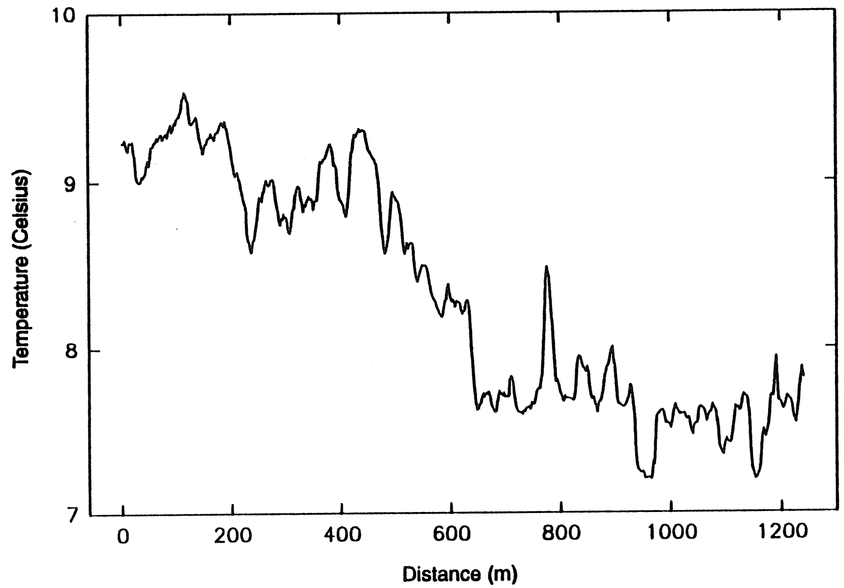

Figure 7-7. Horizontal temperature profile at 30-m depth in Lake Ontario, November 1991.

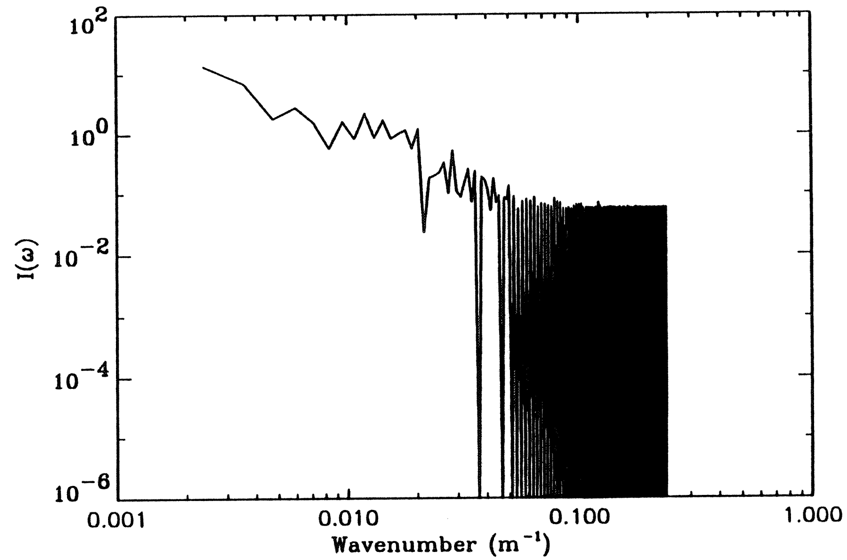

Figure 7-8. Raw periodogram of horizontal temperature profile from Lake Ontario shown in Figure 7-7.

contingency periodogram for five categories (Figure 7-9A) identifies a minor peak at approximately 294 m. The contingency periodogram for the binary series (Figure 7-9B) shows a very strong peak centered around 357 m, also corresponding to the long-term trend in the quantitative data.

The long-term trend is detected in each of the semi-quantitative series, although

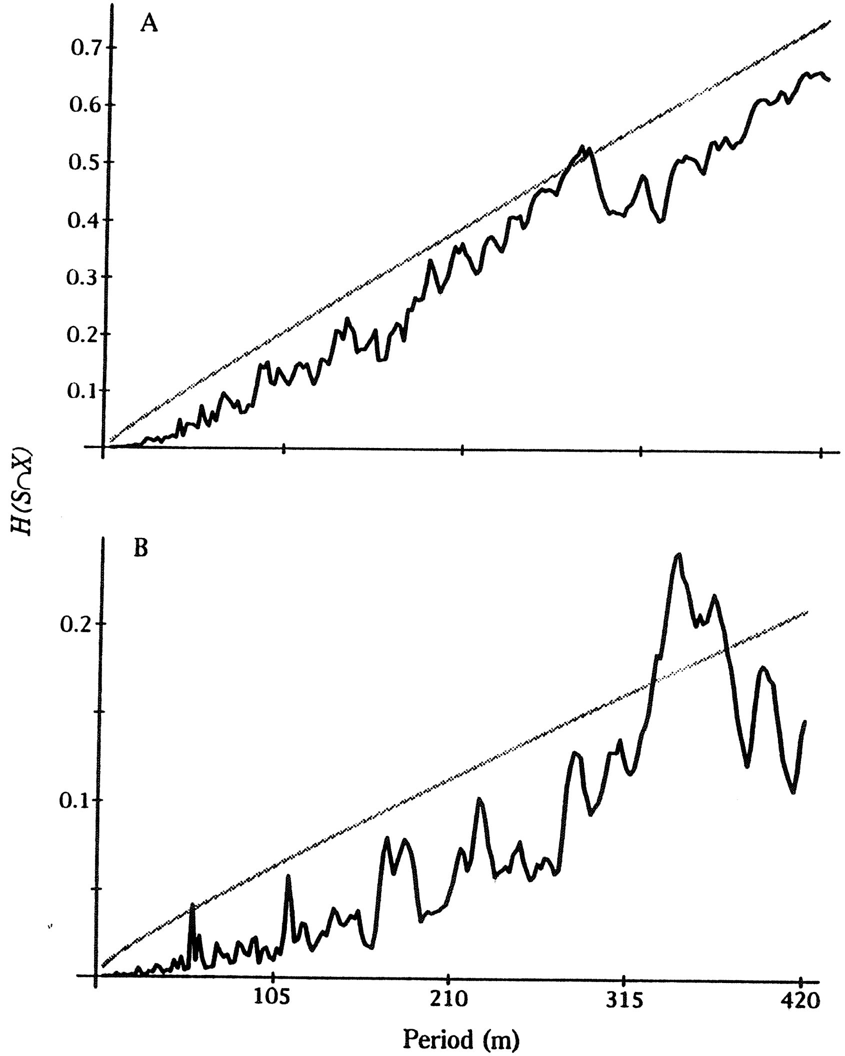

Figure 7-9. Contingency periodograms for the horizontal temperature profile from Lake Ontario as (A) five categories and (B) two categories.

the peak is not very strong in the five-category case. A shift from five to two categories shifts the peak of the contingency periodogram from 294 to 357 m. This raises an important question: how does categorizing affect results and their subsequent interpretation? More important, do the number of categories and the partition values between categories properly "arrange" the given data to give results that are meaningful to the questions being asked? Basically, this technique consists of matching the scale of the sampling with the scale of the question (as shown in the following example).

Example 4: Application to Marine Biology

The salp (*Thalia democratica*) is a gelatinous zooplanktonic filter feeder. Its reproductive strategy is such that a few individuals may give rise to quick population bursts (blooms) in time and space. This generally occurs in the spring. Samples were collected weekly from 1967 to 1990 at a fixed station (Bay of Villefranche-sur-Mer, western Mediterranean) with vertical net hauls. Abundance classes were approximately based upon a geometric progression with base 4.3 (Table 7-3), as determined by Frontier (1969). Missing values are coded as −1. Frontier (1969) showed that this semi-quantitative determination is well suited for describing the spatial and temporal variations of zooplanktonic species at classical regional scales in oceanographic studies.

The semi-quantitative data capture the salient features of the series: strong seasonality with periods of few or no salp, high year-to-year variability with respect to intensity and duration of blooms, and the presence of missing values (Figure 7-10). Note that the sampling design is not well suited for precise determination of the size of the population: salp are distributed in swarms, and vertical sampling at a fixed station precludes the evaluation of spatial fluctuations. *We are interested here in the possible occurrence of blooms, which can be detected by this semi-quantitative measure.*

To reduce the salp series to a binary series, we use biological criteria. We consider that fluctuations between classes 1 and 2 in the original series are attributable to sample fluctuations. Therefore, those two classes form the first category. The second category consists of classes 3 to 5 in the original series, which correspond to a higher concentration of salp (the possible occurrence of a bloom).

Table 7-3. Abundance classes (categories) for the salp time series in Figure 7-10.

Number of salp per haul	Abundance class
0	1
1 to 3	2
4 to 17	3
18 to 80	4
> 80	5

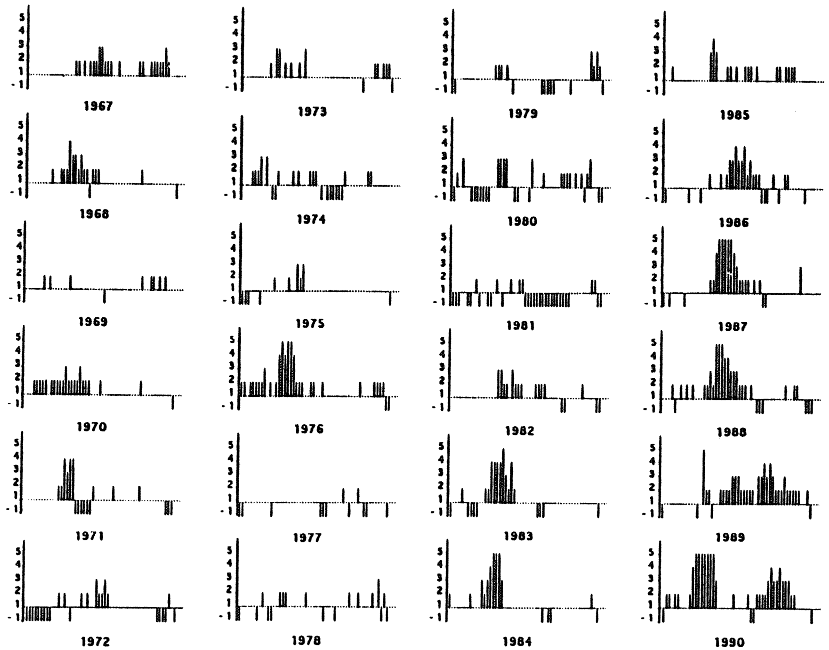

Figure 7-10. The salp time series from 1967 to 1990 at a Mediterranean reference station. Missing values are coded as −1. (Reprinted from F. Ménard, S. Dallot, and G. Thomas, 1993, A stochastic model for ordered categorical time series. Application to planktonic abundance data, *Ecological Modelling* 66:101–112. By permission of Elsevier Science Publishers.)

In the five-category case, a very weak periodicity exists at 52 weeks (Figure 7-11A), signifying a possible yearly cycle. In the two-category case, a strong peak occurs at week 52, as well as later harmonics (Figure 7-11B). A slight peak is also evident at week 26, indicating a possible semi-annual cycle in salp blooms.

It seems obvious that the abundance of salp at time *t* depends on the abundance at previous times. Ménard et al. (1993) have shown, using a periodic Markov model, that significant information is contained in the abundance of the previous week. Considering this, an alternative approach might be to base the statistic $H(S \cap X)$ on transition probabilities (e.g., the probability that salp abundance changes from category *i* to category *j* over one unit of time). However, the advantage of the approach by Legendre et al. is that it does not require the dependence pattern of the series to be explicitly modeled. Its asymptotic distribution will be χ^2 as long as the series can be considered ergodic and stationary under the null hypothesis of no periodicity.

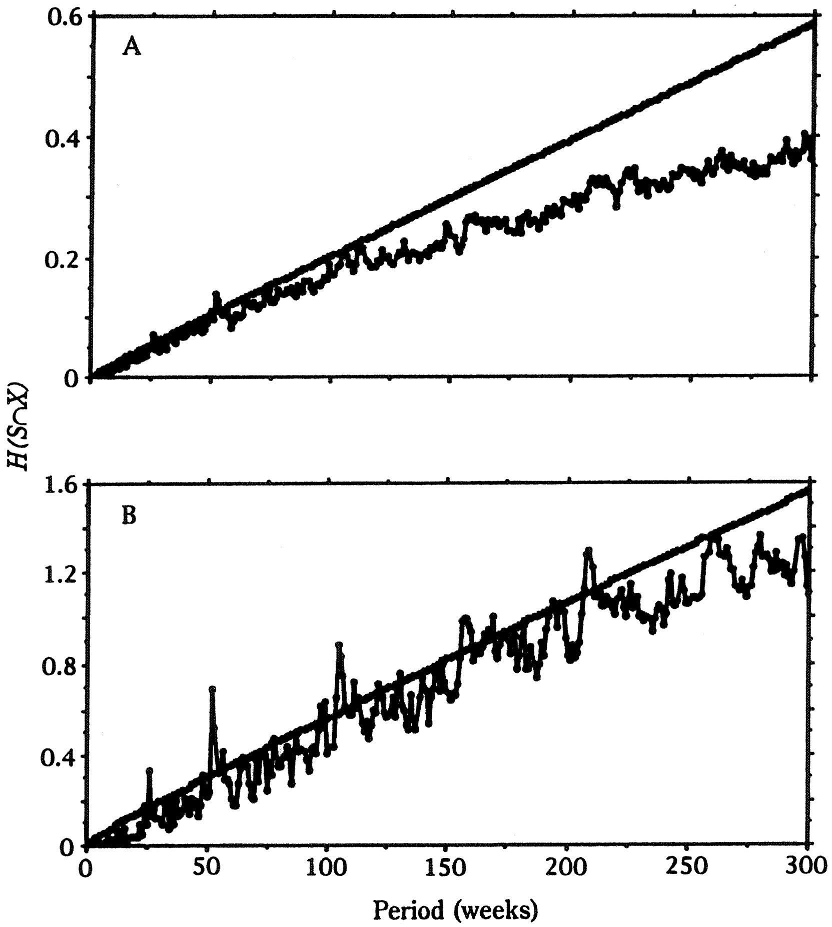

Figure 7-11. The contingency periodograms for the salp time series as (A) five categories (original series) and (B) two categories.

Conclusion

The use of time series analysis in ecology has been increasing in recent years. Its application to ecological data is appropriate and useful when sufficiently long, quantitative datasets are available. In many instances, however, semi-quantitative time series are the norm, and few methods have been proposed to analyze these types of time series for periodicities.

Here we have reviewed the use of one such method, the contingency periodogram, to investigate the ability to detect periodicities when time series are reduced from quantitative to semi-quantitative data. In general, the results are quite consistent: periodicities (or lack of periodicities) are preserved despite decreases in data resolution (Examples 1, 2, 3). For the salp series (Figure 7-10), the contingency periodogram identified a yearly periodicity in blooms in both the original (five-category) and the binary series (Figure 7-11). The $H(S \cap X)$ values in this example appear suspect, likely because of the strong autocorrelation within the dataset, and so have been adjusted by accounting for first-order dependence. We suggest caution when applying the contingency periodogram to field data. Appropriate checks must be made to ensure that short-term serial dependence or long-term trends are not obscuring or biasing results.

As the resolution in a dataset decreases, the ability to detect finer-scale structure also decreases (Example 1). However, as the contingency periodogram does not take into account the ordering of categories (e.g., a change from category 1 to category 2 is considered the same as a change from category 1 to category 4), there may be a loss of information (in terms of contingency periodogram analysis) when the number of groups is *increased*. This appears to be the case for the temperature data (Figure 7-7), where the contingency periodogram shows a much stronger peak in the binary form than in the five-category form.

There is also a discrepancy in the location of the peak in the contingency periodogram between five versus two categories for the temperature data (Figure 7-9). The researcher must decide how much discrepancy can be tolerated without losing the ability (or at least the potential) to answer the question(s) being asked. For instance, in the seasonal model (Figure 7-2), if we are interested in dynamics occurring at the temporal scale of years, then sampling with two categories appears to be sufficient (and more economical). If we are interested in seasonal dynamics, however, then an increase in the number of categories (and an increase in costs) is more appropriate. If we are interested in looking at weekly or monthly dynamics, then some number of categories ≥ 5 (all the way up to quantitative measures) is more appropriate. At the same time, sampling frequency should increase as well, to examine shorter time-scale fluctuations. In the salp analysis (Example 4), the categorization of the data was sufficient to help answer the question we were asking. When the number and partition values of categories do not properly reflect the questions being addressed, the effects can be deleterious. Further research on the effects of differing partition values is needed.

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